Manipulator Dynamics

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MMAE 540 – Introduction to Robotics
The form of the equations of motion for a 2-link planar manipulator are:

$$H(q) \ddot{q} + C(\dot{q}, q) \dot{q} + g(q) = \tau + "\text{other}"$$

- Inertia Matrix
- Coriolis and Centripetal Matrix
- Gravity
- Input Torques
- Additional Disturbance Torques (e.g. viscous friction)
\[ \mathbf{H}(\mathbf{q}) \ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) \dot{\mathbf{q}} + \mathbf{g}(\mathbf{q}) = \tau \]

\[ \mathbf{q} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} H_{11} & H_{12} \\ H_{12} & H_{22} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} -h \dot{\theta}_2 & -h \dot{\theta}_1 - h \dot{\theta}_2 \\ h \dot{\theta}_1 & 0 \end{bmatrix}, \quad \mathbf{g} = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} \]

- Moment of inertia of link 1 with respect to joint 1
- Moment of inertia of link 2 with respect to joint 1
- The square of the distance from the center of mass of the second link to joint 1
- Moment of inertia of the second link with respect to joint 2
- Coupling terms of the moments of inertia
- Coriolis term
- Centripetal terms
- The torque caused by the mass of link 1, acting on joint 1
- The torque caused by gravity of link 2 acting on joint 1
- The torque caused by gravity of link 2 acting on joint 2

\[ H_{11} = m_1 l_{c1}^2 + I_1 + m_2 (l_1^2 + l_{c2}^2 + 2l_1 l_{c2} \cos \theta_2) + I_2 \]

\[ H_{22} = m_2 l_{c2}^2 + I_2 \]

\[ H_{12} = m_2 l_1 l_{c2} \cos \theta_2 + m_2 l_{c2}^2 + I_2 \]

\[ h = m_2 l_1 l_{c2} \sin \theta_2 \]

\[ G_1 = m_1 l_{c1} g \cos \theta_1 + m_2 l_{c2} g \cos (\theta_1 + \theta_2) + m_2 l_1 g \cos \theta_1 \]

\[ G_2 = m_2 l_{c2} g \cos (\theta_1 + \theta_2) \]
Manipulator Dynamics Mini-Quiz

• Describe the components of the C matrix